For typical organic synthesis processes, where bubbling is used, $t_{\text{reg}} \approx 0.02$ sec and the condition of applicability is satisfied for $t_{\text{diem}} > 0.03$ sec.

From an analysis of the problem (13), it follows that for slow reactions, when only the linear term in the expansion with respect to λ is taken into account, Eq. (19) has the same form for a first-order reaction. It is only necessary to carry out the formal substitution for the quantity $\varkappa$, setting $Bk \rightarrow k'$ and $\varkappa \rightarrow k'd^2/4\mathcal{D}$.

NOTATION

Here a is a constant; A, concentration of substance 1 on the bubble surface; B, concentration of substance 2 far away from the bubble; C, concentration of substance 1; C', concentration of substance 2; D^ν, symbol for fractional differentiation; d, diameter of the bubble; F, total mass flux through the bubble surface; f, arbitrary function; I, J, functions of the angles of separation of the flow, entering into the solution; k, rate constant of the second-order reaction; k', rate constant of the first-order reaction; Q, source function of the substance; q_s, gradient of the dimensionless concentration at the boundary of the region; R, bubble radius; r, radial coordinate; f_{chem}, characteristic time of the chemical reaction; t_{reg}, characteristic time for regenerating the bubble surface; u_r, u_θ, radial and angular components of the fluid velocity; z, an integration variable; α, β, constants; δ, variable related linearly to the coordinate; η, integration variable; θ, ϑ, polar coordinates; $\varkappa$, dimensionless rate constant of the chemical reaction; σ, σ', dimensionless concentrations of components 1 and 2; ξ, τ, dimensionless coordinates; φ, angle of separation of the flow; Pe, Peclet number; Sh, Sherwood number; Re, Reynolds number. Indices: s, surface.

LITERATURE CITED

1. Yu. I. Babenko, Solution of inhomogeneous problems in the theory of heat transfer with the help of fractional differentiation," PMM, 38, No. 5, 929-931 (1974).
2. Yu. I. Babenko, "Heat transfer in semiinfinite region with variable physical parameters," Inzh. Fiz. Zh., 34, No. 2, 262-264 (1978).
3. N. N. Rulev, "Hydrodynamics of a rising bubble," Kolloidn. Zh., 42, No. 2, 252-263 (1980).
4. B. I. Brounshtein and G. A. Fishbein, Hydrodynamics, Mass, and Heat Transfer in Dispersed Systems [in Russian], Khimiya, Leningrad (1977).

DETERMINATION OF COEFFICIENT OF EXTERNAL MASS TRANSFER IN DRYING PROCESSES

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A method is proposed for determining the coefficient of external mass transfer from experimental curves of drying kinetics in porous materials. A comparison of results obtained by this method and by the method of moisture content measurements indicates a close agreement.

Important aspects of studying the mass transfer in systems with a solid phase are gathering of experimental data on the mass transfer coefficients, and related with it, development of methods of determining their dependence on the concentration of fluid substance in the porous body. The coefficient of external mass transfer, referred to the motive force in the solid phase, determines the intensity of transfer of the bounded substance from the surface of a capillary-porous body to the ambient medium during a drying process, and can be calculated from the relation

$$j = \beta_{\bar{u}} (u_s - u_e), \tag{1}$$

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where j is the moisture flux. The quantity $\beta_{\bar{u}}$ in this equation is found from the drying intensity by the transient method. The main difficulty here is determining the surface moisture content u_s in the material. The moisture content in the surface layer appears in that equation and it is indeed the thickness of this layer, the governing quantity in this method, which can be selected arbitrarily by each researcher depending on the properties of the dried material as well as on the conditions of the experiment. This arbitrariness introduces an appreciable inaccuracy into the determination of mass transfer coefficients and is the main cause of the discrepancy between data obtained by various authors. The proposed method of determining $\beta_{\bar{u}} = f(\bar{u})$ belongs in the class of transient methods, but it does not require an experimental determination of the local moisture concentration at the surface of the solid body and can be used for determining the external mass transfer of fine-layer or granular materials in study of drying, adsorption, and extraction processes. It is based on a zonal treatment of the drying process, with use of an analytical solution to the system of linear equations of heat and mass transfer [1]. The zonal method of calculation assumes that the kinetic coefficients remain constant over a narrow range of variation of the integral moisture content. The solutions to the equations of mass transfer in terms of mean-volume concentration in bodies of classical shapes with boundary conditions of the third kind, often encountered in practice, are written as

$$\bar{E} = \sum_{n=1}^{\infty} \frac{2N_{\mathrm{Bi,m}}}{\mu_n^2 (N_{\mathrm{Bi}^2,\mathrm{m}} + N_{\mathrm{Bi,m}} + \mu_n^2)} \exp(-\mu_n^2 N_{\mathrm{Fo}}), \tag{2}$$

for a plate,

$$\bar{E} = \sum_{n=1}^{\infty} \frac{4\, N_{\mathrm{Bi}^2,\mathrm{m}}}{\mu_n^2 (\mu_n^2 + N_{\mathrm{Bi}^2,\mathrm{m}})} \exp(-\mu_n^2 N_{\mathrm{Fo}}) \tag{3}$$

for a cylinder, and

$$\bar{E} = \sum_{n=1}^{\infty} \frac{6\, N_{\mathrm{Bi}^2,\mathrm{m}}}{\mu_n^2 (\mu_n^2 + N_{\mathrm{Bi}^2,\mathrm{m}} + N_{\mathrm{Bi,m}})} \exp(-\mu_n^2 N_{\mathrm{Fo}}). \tag{4}$$

for a sphere. An analysis of the transient preceding the regular process mode in terms of mean-volume concentration [2] has revealed that the first term of Eqs. (2)-(4) suffices for calculation of the kinetics. The degree of regularization will then be characterized by the error due to disregarding all other terms of the series, this error being estimated for various values of the Biot number as

$$\varepsilon = \frac{\bar{E}_1 - \bar{E}_\infty}{\bar{E}_1}\, 100\,\%. \tag{5}$$

We will now demonstrate how the coefficient of external mass transfer $\beta_{\bar{u}}$ can be found from the equations of the regular mode, when the relations $\bar{E} = f(N_{\mathrm{Fo}})$ and $a/_{\mathrm{m}} = f(\bar{u})$ are known. The corresponding characteristic equations for Eqs. (2)-(4) are

$$\operatorname{ctg} \mu_1 = \frac{\mu_1}{N_{\mathrm{Bi,m}}}; \quad \frac{J_0(\mu_1)}{J_1(\mu_1)} = \frac{\mu_1}{N_{\mathrm{Bi,m}}}, \quad \operatorname{tg} \mu_1 = \frac{\mu_1}{N_{\mathrm{Bi,m}} - 1}. \tag{6}$$

It is possible [3] to establish the relation $\mu_1 = f(N_{\mathrm{Bi,m}})$ in various ranges of the Biot number $N_{\mathrm{Bi,m}}$. For $N_{\mathrm{Bi,m}} \leq 0.1$ and an infinitely large plate we have

$$\mu_1^2 = N_{\mathrm{Bi,m}} \tag{7}$$

Inserting this into Eq. (2) and letting h = 1, we obtain the expression

$$\bar{E}_1 = \frac{2}{N_{\mathrm{Bi,m}} + 2} \exp(-N_{\mathrm{Bi,m}} N_{\mathrm{Fo}}). \tag{8}$$

For $N_{\mathrm{Bi,m}} > 0.1$ the function $\mu_1^2 = f(N_{\mathrm{Bi,m}})$ becomes

$$\frac{(\mu_1^2)_\infty}{\mu_1} - 1 = \frac{A}{N_{\mathrm{Bi}}{}^{k}{}_{,\mathrm{m}}}. \tag{9}$$

When $N_{\mathrm{Bi,m}} \to \infty$, then $\mu_n = (2n - 1)\,\pi/2$, $\mu_1^2 = \pi^2/4$. Then

$$\frac{\pi^2}{4\mu_1^2} - 1 = \frac{A}{N_{\mathrm{Bi}}{}^{k}{}_{,\mathrm{m}}}. \tag{10}$$

The values A = 2.24 and k = 1.02 for a plate are found from the graph of the relation [3]

$$\lg\left[\frac{(\mu_1^2)_\infty}{\mu_1} - 1\right] = f(\lg N_{\mathrm{Bi,m}}) \tag{11}$$

Inserting expression (11) into Eq. (2), including these found values of A and k, we obtain the expression

$$\overline{E}_1 = \frac{2N_{\mathrm{Bi}^2,\mathrm{m}}}{\frac{\pi^2}{4}\left[\frac{1}{1+2.24/N_{\mathrm{Bi,m}}^{1.02}}\right]\left\{N_{\mathrm{Bi,m}}^2 + N_{\mathrm{Bi,m}} + \frac{\pi^2}{4}\left[\frac{1}{1+2.24/BN_{\mathrm{Bi,m}}^{1.02}}\right]\right\}} \exp\left(-\frac{\pi^2}{4}\left[\frac{1}{1+2.24/N_{\mathrm{Bi,m}}^{1.02}}\right]N_{\mathrm{Fo}}\right). \tag{12}$$

for calculating $N_{\mathrm{Bi,m}}$ and $\beta_{\overline{u}}$. For an infinitely long cylinder and for a sphere with $N_{\mathrm{Bi,m}} \leq 0.1$ we have, respectively

$$\mu_1^2 = N_{\mathrm{Bi,m}};\ \mu_1^2 = 3N_{\mathrm{Bi,m}}. \tag{13}$$

On the basis of relations (13), the equations for calculating $N_{\mathrm{Bi,m}}$ in the regular mode become

$$\overline{E}_1 = \frac{2}{2+N_{\mathrm{Bi,m}}} \exp(-2N_{\mathrm{Bi,m}}N_{\mathrm{Fo}}) \tag{14}$$

for a cylinder and

$$\overline{E}_1 = \frac{2}{2+N_{\mathrm{Bi,m}}} \exp(-3N_{\mathrm{Bi,m}}N_{\mathrm{Fo}}) \tag{15}$$

for a sphere.

Calculations reveal that in the $N_{\mathrm{Bi,m}} > 0.1$ range for a cylinder A = 2.45 and k = 1.04, for a sphere A = 2.70 and k = 1.07, and the roots of the characteristic equations are

$$\mu_1^2 = (2.405)^2\left(\frac{1}{1+2.45/N_{\mathrm{Bi,m}}^{1.04}}\right), \tag{16}$$

$$\mu_1^2 = \pi^2\left(\frac{1}{1+2.7/N_{\mathrm{Bi,m}}^{1.07}}\right). \tag{17}$$

On the basis of expressions (16) and (17), the equations for $N_{\mathrm{Bi,m}}$ in the $N_{\mathrm{Bi,m}} > 0.1$ range become

$$\overline{E}_1 = \frac{4N_{\mathrm{Bi}^2,\mathrm{m}}}{(2.405)^2\left(\frac{1}{1+2.45/N_{\mathrm{Bi,m}}^{1.04}}\right)\left[(2.405)^2\left(\frac{1}{1+2.45/N_{\mathrm{Bi,m}}^{1.04}}\right) + N_{\mathrm{Bi}^2,\mathrm{m}}\right]} \exp\left[-2.405^2\left(\frac{1}{1+2.45/N_{\mathrm{Bi,m}}^{1.04}}\right)N_{\mathrm{Fo}}\right], \tag{18}$$

for a cylinder

$$\overline{E}_1 = \frac{6N_{\mathrm{Bi}^2,\mathrm{m}}}{\pi^2\left(\frac{1}{1+2.7/N_{\mathrm{Bi,m}}^{1.07}}\right)\left[\pi^2\left(\frac{1}{1+2.7/N_{\mathrm{Bi,m}}^{1.07}}\right) + N_{\mathrm{Bi}^2,\mathrm{m}} - N_{\mathrm{Bi,m}}\right]} \exp\left[-\pi^2\left(\frac{1}{1+2.7/N_{\mathrm{Bi,m}}^{1.07}}\right)N_{\mathrm{Fo}}\right]. \tag{19}$$

for a sphere.

With the aid of the thus obtained Eqs. (10), (12), (14), (15), (17), and (18) it is possible to determine from experimental drying curves the values of the Biot mass-transfer number and thus also of $\beta_{\overline{u}}$ in various ranges of moisture concentration. The proposed method of determining $\beta_{\overline{u}}$ assumes that the dependence of the molecular diffusivity a_m on the concentration and the temperature is known.

For making the calculations more convenient, in Fig. 1 have been plotted graphs of the relation $\overline{E} = f(N_{\mathrm{Bi,m}})$ on the basis of these equations.

The sequence of determining the mass transfer coefficients is as follows. From the experimental curve of mean-volume concentration one selects the range of concentrations $\overline{E}_i\,(\overline{u}_f - \overline{u}_e)/(\overline{u}_i - \overline{u}_e)$ within the time interval $\Delta\tau_i$ in which the kinetic coefficients are assumed to remain constant. For this selected concentration range one determines, from the available $a_m = f(\overline{u})$ curve, the mean value of the molecular diffusivity. Insertion of $\overline{E}_i$ and $N_{\mathrm{Fo}} = a_m\Delta\tau_i/R^2$ into the corresponding equation yields the mass-transfer number

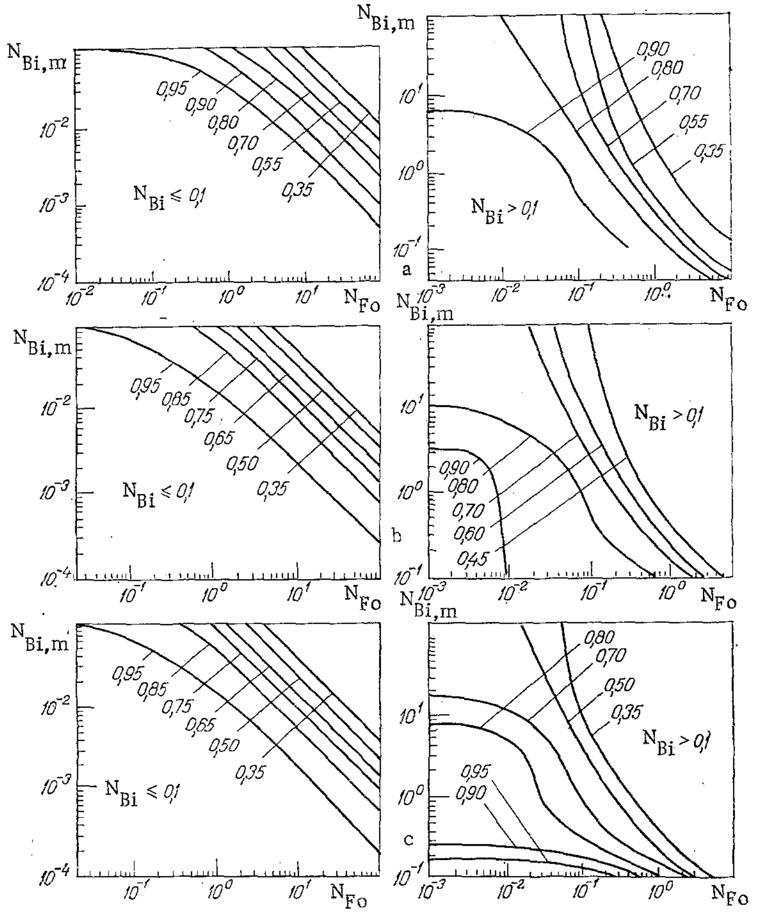

Fig. 1. Dependence of $N_{Bi,m}$ on N_{Fo} for (a) infinitely large plate, (b) cylinder, and (c) sphere; numbers at the curves indicate the values of E.

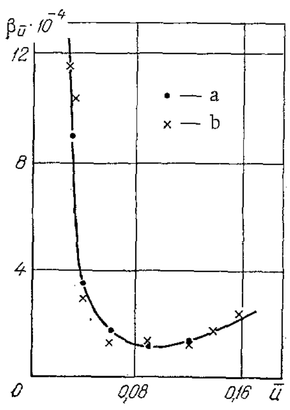

Fig. 2. External mass transfer coef. $\beta_{\bar u}$(m/sec) as a function of the moisture content $\bar u$ (kg/kg) according to the method of moisture content measurement (a) and according to the proposed method (b).

$$N_{Bi,m} = \frac{\beta_{\bar u}}{a_m} R, \tag{20}$$

and thus the coefficient of external mass transfer. Then, from the $\varepsilon = f(\bar E, N_{Bi,m})$ graph [2], one estimates the error of $N_{Bi,m}$ calculation by the zonal method. Assuming a constant $\beta_{\bar u}$ within the $\bar E$ interval, one plots the calculated $\beta_{\bar u}$ in $\beta_{\bar u} - \bar u$ coordinates (Fig. 2). Successive calculations by precisely the procedure for next intervals of $\bar u$ and τ yield the concentration dependence of the external mass transfer coefficient. A comparison of

$\beta_{\bar{u}}$ values in Fig. 2 for drying of steatite ceramic determined from the drying intensity according to Eq. (1) and according to our method respectively indicate a close agreement.

The question as to which equation has to be used for determining the Biot number in a given range of moisture content variation can be answered uniquely in each specific case. When Eq. (10) yields a Biot number $N_{Bi,m} = 0.1$, for instance, then Eq. (12) must be used for calculations, and so on in the case of other bodies. In the case of drying in a fluidized disperse coolant, the Biot number will exceed 0.1 very soon after the beginning of the process [4]. The mass transfer coefficients must then be calculated according to Eqs. (12), (14), and (15).

NOTATION

$\beta_{\bar{u}}$, external mass transfer coefficient referred to the motive force in the solid phase; u_s, u_e, respectively, the surface moisture content and the equilibrium moisture content in the porous body; $N_{Bi,m} = \beta_{\bar{u}}R/a_m$, Biot mass-transfer number; $N_{Fo} = a_m\tau/R^2$, Fourier number; a_m, molecular diffusivity; τ, time; R, characteristic dimension of the body; μ_n, roots of the characteristic equations; and u, mean-volume moisture content.

LITERATURE CITED

1. A. V. Lykov, Theory of Drying [in Russian], Énergiya, Moscow (1968).
2. A. N. Planovskii, S. P. Rudobashta, and G. N. Kormil'tsin, "Determining the ranges of regular mode in molecular diffusion problems," Teor. Osn. Khim. Tekhnol., 6, No. 3, 459-462 (1972).
3. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
4. E. N. Prozorov, "Duration of heat treatment of products made of engineering-grade ceramic in fluidized bed," Elektrotekh. Promyshl., No. 7, 14-16 (1975).

HEATING OF CERAMIC AND SILICATE MATERIAL SURFACES BY AN ARC PLASMA FILAMENT

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The heating of ceramic and silicate material surfaces by a heavy current arc plasma filament is investigated in an air atmosphere. The possibility of using optical pyrometry methods for highly heated surfaces is studied for $T > 2000°K$.

A rise in the efficiency of many high-temperature technological processes requires a detailed study of the interaction between high-enthalpy plasma fluxes and the surface of a solid. Despite the considerable amount of investigations, there are many unsolved questions in this area [1]. Especially few are the confident results in studying plasma interaction with a surface with ablation taken into account. Theoretical methods do not permit the confident solution of this problem at this time because of the complexity of taking account of the whole set of elementary processes occurring here, as well as the lack of information about the properties of a plasma that are realized in the boundary zone during interaction. Difficulties in the experimental investigations are due primarily to the lack of reliable methods and apparatus for body temperature and plasma parameter diagnostics during its action on the body surface.

The behavior of ceramic and silicate materials subjected to plasma fluxes is investigated in this paper, and the possibility of using optical pyrometry methods for a heated surface is examined. The schematic diagram is presented in Fig. 1a, and is described partially in [2]. In order to assure high density of the energy delivered stationarily to the item surface and to obtain reproducible results, a current-carrying plasma filament placed between two specimens being investigated which were mounted in parallel at a 6-mm spacing and

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